PROCEEDINGS

OF

THE ROYAL SOCIETY.

December 18, 1879.

THE PRESIDENT in the Chair.

The Presents received were laid on the table, and thanks ordered for them.

The following Papers were read:—

I. "On the Secular Changes in the Elements of the Orbit of a Satellite revolving about a Tidally Distorted Planet." By G. H. DARWIN, F.R.S. Received December 8, 1879.

(Abstract.)

This memoir is a continuation of four previous papers on similar subjects.*

As the investigation is entirely analytical, and is rather long and complicated, it seems useless to attempt a detailed abstract, and I shall therefore confine myself to giving an outline of the results, and a few remarks on the method employed. I also here partially replace the analytical treatment of the paper itself by general reasoning, so as to give some idea of the physical causes which underlie the definite results of analysis. These general considerations are not, however,

* 1. "On the Bodily Tides of Viscous and Semi-elastic Spheroids, and on the Ocean Tides upon a Yielding Nucleus."

2. "On the Precession of a Viscous Spheroid, and on the Remote History of the Earth."

3. "On Problems connected with the Tides of a Viscous Spheroid."

These three will appear in the "Phil. Trans." for 1879.

4. "The Determination of the Secular Effects of Tidal Friction by a Graphical Method." "Proc. Roy. Soc.," vol. xxix, 1879.

strictly appropriate to an abstract, since they do not occur in the paper, and merely serve as a rough substitute for analysis.

In this and the previous papers it is supposed that tides are raised in a planet by its satellites, and the problem is to determine the various effects which result from the friction of those tides.

The hypothesis generally adopted in these papers is that the planet is a viscous body, and that the tides are a bodily distortion of the whole mass of the planet, but nearly all the results would also follow from the friction of oceanic tides upon a rigid nucleus.

The investigation is principally directed towards the case of the earth, sun, and moon, and the phraseology of the paper is taken from our own planet and satellite; but the methods may be extended to the other planets.

The subject will be most easily explained by inverting the order of the paper, and by beginning with the sketch of the results.

Sketch of Results.

At the present time the moon revolves round the earth in 27·3 days. The orbit has an eccentricity of $\frac{1}{18}$, and is inclined at an angle of 5° 9′ to a certain plane, which is said to be "proper to the orbit." This proper plane is inclined to the ecliptic at an angle of about 8″, and intersects the ecliptic in the equinoctial line; it lies on the same side of the ecliptic as the earth's equator.

In this statement the "periodic inequalities" of the moon's motion are neglected.

In this and the previous papers it is proved that frictional tides in the earth are causing, and must have caused, changes in the configuration of the system. The changes in the past may be summarised as follows:—

1. The lunar period must have been shorter in the past, and may be traced back from the present 27·3 days, until initially the moon revolved round the earth in from 2 to 4 hours.

2. The inclination of the orbit to the proper plane must have been larger in the past, and may be traced back from the present 5° 9′ until it was 6° or 7°. This 6° or 7° was a maximum inclination, and in the more remote past the inclination was less, and initially was very small, or zero.

3. The inclination of the proper plane to the ecliptic must have been greater in the past, and may be traced back from the present 8″, until it was in very early times about 11° 45′. It is possible that initially this inclination was less, and that the 11° 45′ of inclination was a maximum value.

4. The eccentricity of the orbit must have been smaller in the past. Either at one time it had a minimum value, before which it had a

maximum value, and again earlier it was very small, or zero; or else the maximum value never occurred, and the eccentricity has always been increasing. The history of the eccentricity depends on the nature of the tides in the earth, but the former of these alternatives seems the more probable.

We will now consider the earth.

At the present time the earth rotates in 24 hours, its equator is inclined at an angle of about 9″ to a plane, which is called in this paper "the proper plane of the earth." This proper plane is inclined at an angle of 23° 28′ to the ecliptic, and its intersection with the ecliptic is the equinoctial line.

(In the ordinary mode of statement the proper plane is called the mean equator, and the true equator is described as nutating about the mean equator with a period of 19 years, and an amplitude of 9″.)

It is here proved that the frictional tides in the earth have caused changes, which may be summarised as follows:—

5. The earth's period of rotation, or the day, must have been shorter in the past, and it may be traced back from the present value of 24 hours, until initially it was from 2 to 4 hours in length. It was then identical with the moon's period of revolution as described in (1).

6. The inclination of the equator to "the earth's proper plane," must have been larger in the past, and may be traced back from the present value of 9″, until it was about 2° 45′. This 2° 45′ was a maximum inclination, and in the more remote past the inclination was less, and initially it was very small, or zero.

7. The inclination of "the earth's proper plane" to the ecliptic must have been smaller in the past, and may be traced back from its present value of 23° 28′, until initially it was 11° 45′, or perhaps somewhat less. It was then identical with the proper plane of the lunar orbit; and this is true whether or not 11° 45′ was a maximum inclination of the lunar proper plane to the ecliptic, as described in (1).

The preceding statements may be subject to varieties of detail, according to the nature of the tides raised in the earth, but the above is a summary of what appears to be the most probable course of evolution.

The hypothesis which is suggested as most probable is, that the more recent changes in the system have been principally due to oceanic tidal friction, and that the more ancient changes were produced by bodily tidal friction.

These seven statements, when taken together, exhibit the earth and moon initially nearly in contact; the moon always opposite the same face of the earth, or moving very slowly relatively to the earth's surface; the whole system rotating in from 2 to 4 hours, about an axis inclined to the normal to the ecliptic at an angle of 11° 45′, or some-

what less; and the moon moving in a circular orbit, the plane of which is nearly coincident with the earth's equator.

This initial configuration suggests that the moon was produced by the rupture, in consequence of rapid rotation or other causes, of a primeval planet, whose mass was made up of the present earth and moon. The coincidence is noted in the paper, that the shortest period of revolution of a fluid mass of the same mean density as the earth, which is consistent with an ellipsoidal form of equilibrium, is 2 hours 24 minutes; and that if the moon were to revolve about the earth with this periodic time, the surfaces of the two bodies would be almost in contact with one another.

Tidal friction is a *vera causa*, and the only postulates of this theory of the evolution of our system are lapse of time, and the non-existence of sufficient diffused matter to materially affect the motions of the moon and earth through space.

The systems of the other planets of the solar system are reviewed from the point of view of this tidal theory of evolution, and it is found that there are many confirmatory circumstances, and none which appear condemnatory. But as the present investigation only treats of a planet with a single satellite, it necessarily leaves many points untouched. In relation to this theory, the most interesting points are the satellites of Mars, and the inclinations of the orbits of Jupiter's satellites to their proper planes.

Notes on the Method of Investigation—General Reasoning in Substitution for Analytical Treatment.

The following are the titles of the several parts into which the paper is divided:—

I. The theory of the disturbing function.

II. Secular changes in the inclination of the orbit of a satellite.

III. The proper planes of the satellite, and of the planet, and their secular changes.

IV. Integration for changes of the inclination of the orbit, and of the obliquity of the ecliptic.

V. Secular changes in the eccentricity of the orbit of a satellite.

VI. Integration for changes of eccentricity of the orbit.

VII. Summary and discussion of results.

VIII. Review of the tidal theory of evolution as applied to the earth, and the other members of the solar system.

These titles indicate the method of treatment.

The application of the method of the disturbing function to the present problem has certain peculiarities. The attraction of the tides raised in the planet by a satellite is the cause of the perturbation of the satellite's motion. Now the state of tidal disturbance of the

planet depends upon the position of the satellite, therefore the elements of the satellite's orbit will appear in the disturbing function, as representing the state of tidal disturbance in the planet; but these elements also appear as representing the position of the satellite as a body whose motion is perturbed. In order to apply the method of the disturbing function they ought only to enter in the latter capacity. This difficulty is overcome by supposing the earth to have two satellites; one the tide-raising satellite (called Diana in the memoir), and the other the perturbed satellite or moon. Then, after the application of the method of the disturbing function, the tide-raising satellite is made identical with the moon; and thus the effect of the lunar tides on the moon's motion is determined. Or else the tide-raising satellite is made identical with the sun, so as to find the effect of the solar tides on the moon. The method of the disturbing function is also applied to determine the perturbations of the earth's rotation, and a similar artifice has to be used, because the earth has to be treated in two capacities, *first* as a body in which tides are raised, and *secondly*, as a body whose rotation is perturbed.

The problem is divided into the two following cases :—

1st. Where the lunar orbit is circular, but inclined to the ecliptic.

2nd. Where the orbit is elliptic, but coincident with the ecliptic.

The previous paper on "Precession," dealt with the mean distance of the moon, and with the rotation of the earth and the obliquity of the ecliptic; therefore, in the present paper the inclination and eccentricity afford the principal topics.

The first of these problems occupies the larger part of the paper. If the satellite and planet be the only bodies in existence, the problem of the inclination is not very complicated.

The following considerations (in substitution for the analytical treatment of the paper) will throw some light on the general effects of tidal friction :—

Suppose the motions of the planet and of its solitary satellite to be referred to the invariable plane of the system. The axis of resultant moment of momentum is normal to this plane, and the component rotations are that of the planet's rotation about its axis of figure, and that of the orbital motion of the planet and satellite round their common centre of inertia; the axis of this latter rotation is clearly the normal to the satellite's orbit. Hence the normal to the orbit, the axis of resultant m. of m., and the planet's axis of rotation, must always lie in one plane. From this it follows that the orbit and the planet's equator must necessarily have a common node on the invariable plane.

If either of the component rotations alters in amount or direction, a corresponding change must take place in the other, such as will keep the resultant m. of m. constant in direction and magnitude.

It appears from the previous papers that the effect of tidal friction

is to increase the distance of the satellite from the planet, and to transfer m. of m. from that of planetary rotation to that of orbital motion.

If then the direction of the planet's axis of rotation does not change, it follows that the normal to the lunar orbit must approach the axis of resultant m. of m. By drawing a series of parallelograms on the same diameter and keeping one side constant in direction, this may be easily seen to be true.

The above statement is equivalent to saying that the inclination of the satellite's orbit will decrease.

But this decrease of inclination does not always necessarily take place, for the previous investigations show that another effect of tidal friction may be to increase the obliquity of the planet's equator to the invariable plane, or in other words to increase the inclination of the planet's axis to the axis of resultant m. of m.

Now if a parallelogram be drawn with a constant diameter, it will easily be seen that by increasing the inclination of one of the sides to the diameter (and even decreasing its length), the inclination of the other side to the diameter may also be increased.

The most favourable case for such a change is when the side whose inclination is increased is nearly as long as the diameter. From this it follows that the inclination of the satellite's orbit to the invariable plane may increase, and also that the case when it is most likely to increase is when the m. of m. of planetary rotation is large compared with that of the orbital motion. The analytical solution of the problem agrees with these results, for it shows that if the viscosity of the planet be small the inclination of the orbit always diminishes, but if the viscosity be large, and if the satellite moves with a short periodic time (as estimated in rotations of the planet), then the inclination of the orbit will increase.

These results serve to give some idea of the physical causes which, according to the memoir, gave rise to the present inclination of the lunar orbit to the ecliptic. For the analytical investigation shows that the inclination of the lunar orbit to its proper plane (which replaces the invariable plane when the solar attraction is introduced) was initially small, that it then increased to a maximum, and finally diminished, and that it is still diminishing.

But the laws above referred to would, by themselves, afford a very unsatisfactory explanation of the inclination of the lunar orbit, because the sun's attraction was found to be a matter of much importance.

It was stated above, that if the viscosity of the planet be small, the inclination of the orbit of the solitary satellite to the invariable plane will always diminish; but when solar influence is introduced, the corresponding statement is not true with regard to the inclination of

the lunar orbit to the proper plane, for during one part of the moon's history, the inclination to the proper plane would have increased, even if the viscosity of the earth had been small.

It does, however, follow, from the analytical investigation, that if the lunar orbit was primitively coincident with the earth's equator, then the present *amount* of inclination of the lunar orbit to the ecliptic (viz., $5^\circ\ 9'$) is not explicable on the hypothesis of small viscosity of the earth, but is explicable if we suppose that the viscosity of the earth has always been large, as it certainly is at present. The theory which gives the most perfect account of the present amount of inclination of the lunar orbit is, that the more recent changes in the system have been principally due to oceanic tidal friction, and the more ancient principally to bodily tidal friction, with a large degree of viscosity of the earth's mass.

The presence of the sun rendered it expedient to divide the problem of the inclination into three cases:—

1st. When the solar influence is large, as it is at present.

2nd. When the solar influence is small, or *nil*. This is the case to which the above general considerations apply.

And the third case is intermediate between the first and second cases.

For the third case the theory of the proper planes of the moon and earth had to be investigated, and the problem resolved itself into the determination of the secular changes of the positions of the two proper planes, and of the inclinations of the planes of motion of the two parts of the system to their respective proper planes.

The questions involved in these three cases are, however, so complex that it does not seem advisable to enter on them in this abstract.

The second of the two problems, that of the eccentricity of the orbit, is also treated by the method of the disturbing function.

The result, for a viscous planet, shows that in general the eccentricity of orbit will increase; but if the obliquity of the planet's equator be nearly 90°, or if the viscosity be so great as to approach perfect rigidity, or if the periodic time of the satellite (measured in rotations of the planet) be short, the eccentricity will slowly diminish.

When the viscosity is small the law of variation of eccentricity is very simple, and it appears that if eleven periods of the satellite occupy a longer time than eighteen rotations of the planet, the eccentricity increases, and *vice versâ*. Hence, in the case of small viscosity, a circular orbit is only dynamically stable if the eleven periods are shorter than the eighteen rotations.

In the history of a single satellite revolving about a planet of small

viscosity, the periods of rotation and revolution start from identity and end with identity; hence the eccentricity rises from zero to a maximum, and ultimately decreases to zero again.

It is also proved, that in the history of a single satellite revolving about a planet of large viscosity, the eccentricity rises very rapidly to a maximum, decreases slowly to a minimum, and then increases again; but the actual degree of viscosity has an important influence on the results.

The following considerations (in substitution for the analytical treatment of the paper) throw some light on the physical causes of these results.

Consider a satellite revolving about a planet in an elliptic orbit, with a periodic time which is long compared with the period of rotation of the planet; and suppose that frictional tides are raised in the planet.

The major axis of the tidal spheroid always points in advance of the satellite, and exercises a force on the satellite which tends to accelerate its linear velocity.

When the satellite is in perigee the tides are higher, and this disturbing force is greater than when the satellite is in apogee.

The disturbing force may, therefore, be represented as a constant force, always tending to accelerate the motion of the satellite, and a periodic force which accelerates in perigee and retards in apogee. The constant force causes a secular increase of the satellite's mean distance and a retardation of its mean motion.

The accelerating force in perigee causes the satellite to swing out further than it would otherwise have done, so that when it comes round to apogee it is more remote from the planet. The retarding force in apogee acts exactly inversely, and diminishes the perigeean distance. Thus, the apogeean distance increases and the perigeean distance diminishes, or in other words, the eccentricity of the orbit increases.

Now consider another case, and suppose the satellite's periodic time to be identical with that of the planet's rotation. Then when the satellite is in perigee it is moving faster than the planet rotates, and when in apogee it is moving slower; hence at apogee the tides lag, and at perigee they are accelerated. Now the lagging apogeean tides give rise to an accelerating force on the satellite, and increase the perigeean distance, whilst the accelerated perigeean tides give rise to a retarding force, and decrease the apogeean distance. Hence in this case the eccentricity of the orbit will diminish.

It follows from these two results that there must be some intermediate periodic time of the satellite, for which the eccentricity does not tend to vary.*

* The substance of the preceding general explanation was suggested to me in con-

But the preceding general explanations are in reality somewhat less satisfactory than they seem, because they do not make clear the existence of certain antagonistic influences.

Imagine a satellite revolving about a planet, and subject to a constant accelerating force, which we saw above would result from tidal reaction.

In a circular orbit a constant tangential force makes the satellite's distance increase, but the larger the orbit the less does the given force increase the mean distance. Now the satellite, moving in the eccentric orbit, is in the apogeean part of its orbit like a satellite moving in a circular orbit at a certain mean distance, but in the perigeean part of the orbit it is like a satellite moving in a circular orbit but at a smaller mean distance; in both parts of the orbit it is subject to the same tangential force. Then the distance at the perigeean part of the orbit increases more rapidly than the distance at the apogeean part. Hence the constant tangential force on the satellite in the eccentric orbit will make the eccentricity diminish. It is not clear from the preceding general explanation, when this cause for decreasing eccentricity will be less important than the previous cause for increasing eccentricity.

The disturbing causes which tend to make the eccentricity diminish are (i) the principal semi-diurnal tide, (ii) the "faster elliptic semi-diurnal tide," (iii) the "elliptic monthly tide." The increase of eccentricity depends entirely on (iv) the "slower elliptic semi-diurnal tide." If the periodic time of the satellite be long, as measured in rotations of the planet, the importance of the tides (i), (ii), and (iv) are as the numbers 4, 1, and 49 respectively, and the importance of (iii) is very small; if the satellite were to move faster, the importance of (iv) would decrease, and that of (i), (ii), and (iii) would increase.

In the outline of results at the beginning of the abstract, it was stated that the periodic times of revolution and rotation of the moon and earth might be traced back to a common period of from 2 to 4 hours. In the paper on "Precession" the common period was found to be a little over 5 hours in length; but that result was avowedly based on a partial neglect of the sun's attraction. In this memoir certain further considerations are adduced, which show that, while the general principle remains intact, yet the common period of revolution of the earth and moon must initially have been shorter than 5 hours to an amount, which is uncertain but is probably large. The period of from 2 to 4 hours is here assigned, because it is mechanically impossible for the moon to revolve about the earth in less than 2 hours, and it is uncertain how the rupture of the primeval planet took place.

versation by Sir William Thomson, when I mentioned to him the results at which I had arrived.

It is hardly possible that such general reasoning, as has been above applied to the two problems of the present paper, could ever have led to a discovery of the laws of change of the system. This kind of consideration is, however, of some interest as throwing light on the definite results already attained by the accurate methods of analysis.

II. "On Buff's Experiments on the Diathermancy of Air." By JOHN TYNDALL, F.R.S. Received December 10, 1879.

Two years ago, Dr. Hofmann drew my attention to a forthcoming paper by Professor Buff, to which he obviously and naturally attached considerable importance. The paper appeared in the "Philosophical Magazine" for December, 1877. Being much occupied at the time with other matters, I merely glanced at its conclusions, and then laid it aside until I should be able to read it carefully, and, if necessary, to examine it experimentally. Last summer, I, for the first time, read the paper through, and I have recently, more than once, repeated its perusal—reflecting on its methods and conclusions, to the best of my ability, wherever they appeared dark to me.

The principal result of the paper is, that a stratum of dry air, 45 millims. thick, "absorbs from 50 to 60 per cent. of the rays of heat which it receives from a source heated to the temperature of boiling water." The experiments whereby I sought to show the absorption of radiant heat, by a stratum of dry air more than thirty times the thickness of that employed by Professor Buff, to be sensibly *nil*, are, at the same time, pronounced "unreliable."

I once ventured te express the opinion that 10 per cent. of the radiation from the earth is absorbed by the aqueous vapour contained in the first 10 feet of air. The late Professor Magnus urged against me at the time, that were so much heat lodged in so thin a stratum, the deposition of dew would be impossible. Urged against the conclusion that not 10 per cent., but 60 per cent. of the earth's radiation is absorbed, not within 120 inches, but within 2 inches of the earth's surface, the argument of Professor Magnus would have serious force. Under the covering assigned to it by Professor Buff, our planet ought never to suffer from rapid nocturnal chill.

Professor Buff's intention, at starting, was to investigate the conduction of heat by gases, and he employed for this purpose an apparatus similar to that of Professor Magnus, the dangers attending the use of which I have frequently pointed out. A glass cylinder, with its lower edge ground level, was mounted on the plate of an air-pump. Cemented on to the open top of the cylinder was a brass vessel with a polished horizontal bottom, which was heated by the